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Inviscid Batchelor-model flow past an airfoil with a vortex trapped in a cavity

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An efficient method of constructing inviscid Batchelor-model flows is developed. The method is based on an analytic continuation of the potential part of the flow into the closed-streamline vortex region. Numerical solutions are presented for Batchelor-model flows past airfoils with cavities. With the airfoil and dividing streamline shape, the eddy vorticity, and the jump in the Bernoulli constant across the eddy boundary given, the program calculates the corresponding cavity shape and the entire flow.

1. Introduction

Inviscid Batchelor-model flow is a plane steady flow of incompressible fluid past a body with vorticity that is constant inside and zero outside the region of closed streamlines. As the eddy boundary is a streamline and the flow is inviscid, the tangential velocity component may be discontinuous at this boundary, while the pressure is continuous. Flows of this type have been of interest since Batchelor (1956 *a*) gave an accurate proof of the theorem first formulated by Prandtl (1904). According to this theorem, if an inviscid flow with a region of closed streamlines is a high-Reynolds-number limit of a viscous flow, then inside that region the vorticity is constant. On the basis of this theorem Batchelor (1956 *b*) proposed his famous model of separated flow past a body. Calculations of various inviscid Batchelor-model flows are described in many papers. In particular it is worth mentioning the recent works of Moore, Saffman & Tanveer (1988), Turfus (1993), Chernyshenko (1993), the first calculation (Shabat 1963) of the flow which became later widely known due to the paper of Sadovskii (1971), and the book by Gol'dshtik (1981) where many of the earlier works were reviewed. Further references on inviscid Batchelor-model flows can be found in these works.

For high-Reynolds-number asymptotics of viscous flows it should be noted, however, that in the case of flows past bluff bodies there are strong objections to use of the Batchelor model (Smith 1979; Chernyshenko 1984). When the self-consistent high-*Re* asymptotics of the flow past a bluff body was calculated (Chernyshenko 1988; Chernyshenko & Castro 1993) it turned out to be not of the Batchelor-model type. In other cases, such as the flow past a cavity, the Batchelor model can probably describe the high-*Re* asymptotics of viscous flows provided that certain severe restrictions on the body shape (Chernyshenko 1991) are satisfied. Otherwise the limiting flow is

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not of the Batchelor-model type. Therefore, to study the Batchelor model, the best formulation is an inverse one: to find a body to which the Batchelor model applies.

Generally, for a body of a given shape there is a two-parameter family of inviscid Batchelor-model flows. Presumably, only one member of this family may be the high-*Re* limit of the corresponding viscous flow. Naturally, this member must be found from an analysis involving viscosity effects: namely, an analysis of the flow in the near vicinity of the separation point and an analysis of the cyclic boundary layer surrounding the eddy are usually expected to give two required conditions. However, in the case of an inverse formulation not only the body shape may be adjusted: for example, blowing or suction within the boundary layer may be applied to change the position of the separation point or the value of the vorticity in the eddy. Therefore it makes sense to calculate the entire family of inviscid flows.

Inviscid Batchelor-model flows are also of interest for another reason. They often have qualitatively correct streamline patterns and, regardless of any asymptotic considerations, can sometimes be convenient for an approximate description of realistic (including turbulent) flows. In particular, the Batchelor model may be a reasonable compromise between accuracy and simplicity for describing flows with trapped vortices, that is with massive vortices remaining in the vicinity of a body instead of being shed periodically (or chaotically) into the wake. Vortices of this type can considerably improve the performance characteristics of airfoils and diffusers, but stabilizing such vortices is a very difficult problem (Wu & Wu 1992; Chernyshenko 1995). Since for vortex stabilization the body shape must be designed specifically, an inverse approach may also be useful in this case.

A distinguishing feature of the general method of constructing inviscid Batchelor-model flows, which is described in what follows, is that part of the body shape is determined from the solution of the problem. The exact formulation of the problem is given below. The particular requirements imposed on the body shape are different for the problems of high-Reynolds-number asymptotics of viscous flow and of stabilization of trapped vortices (and as yet unknown in the latter case). Discussing these requirements is outside the scope of the present paper.

2. Problem formulation

The cavity shape and the flow in figure 1, which illustrates the problem formulation, were calculated by the method explained in detail in the following sections.

Let $W = W(z)$ be a complex potential of the flow past a body. Here, $z = x + \mathrm{i}y$, and x, y are the Cartesian coordinates.

Let an auxiliary variable s be mapped conformally onto z by $z = f(s)$ with $f(s)$ given by an explicit expression. The real axis in the s-plane is mapped by $z = f(s)$ onto a curve, which we will call a support curve. Let us assume that the portion of the body surface between points A and B coincides with a portion of the support curve. The rest of the support curve is shown in figure 1 with a broken line. In other words, this part of the body surface can be described in a parametric form as $x = g(s)$, $y = h(s)$ with real s, where $g(s)$ and $h(s)$ are analytic functions that take real values for real s and can be expressed by explicit formulae. Then $f(s) = g(s) + \mathrm{i}h(s)$. The mathematical formulation of the problem is to find the shape of the cavity which can be made in the body surface such that the following conditions are satisfied:

(*a*) the Batchelor-model flow with a closed streamline region inside the cavity coincides, outside, with the flow described by the complex potential W;

(*b*) the dividing streamline begins at A, ends at B, and is described by the equa-

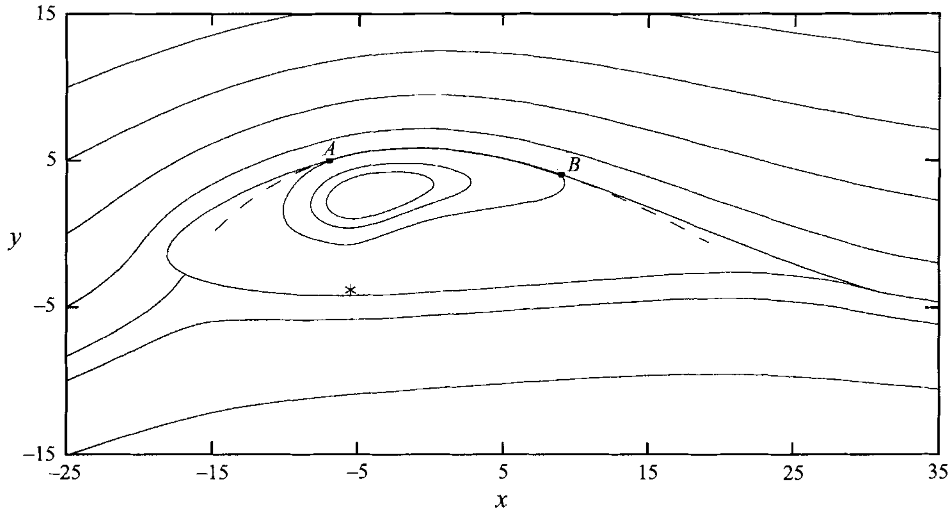

FIGURE 1. Cavity shape and flow streamlines for $\omega = -0.25$ and $\Delta = 1.1$.

tion $z = f(s)$ with real s, that is, it coincides with the previously distinguished portion of the body surface;

(*c*) the vorticity in the cavity and the jump in the Bernoulli constant across the eddy boundary (that is half of the jump in the velocity squared) equal the prescribed values of ω and Δ respectively.

For $\Delta = 0$ and W given by an explicit expression this problem was solved by Abrashkin & Yakubovich (1988).

As shown below, this problem reduces to an analytic continuation, and for this reason it is ill-posed in the sense that an arbitrary small variation in the shape of the dividing streamline may result in a finite change of the cavity shape. That is why the function $f(s)$ was required to be expressed explicitly. It will be seen from the results below, for $f(s)$ fixed, that if the solution exists then it is unique, and small variations in W result in small variations in the cavity shape.

3. General solution and analysis

Let us express the flow inside the cavity as the sum of a potential flow with potential w and a prescribed vortex flow with vorticity ω. If the vorticity is given then the eventual result does not depend on the particular form of the vortex component of the flow. The formulae will be most concise if the vortex component is a rigid-body rotation. Then inside the cavity the flow velocity, with components denoted by u and v, takes the form

$$u - \mathrm{i}v = w'_z - \mathrm{i}\omega\bar{z}, \tag{3.1}$$

where the overbar means, as usual, complex conjugation, and the prime means differentiation with respect to the subscript. Outside the cavity the velocity, with components denoted by U and V, is given by the formula

$$U - \mathrm{i}V = W'_z.$$

On the dividing streamline the velocity is directed along this line and the squares of the velocity inside and outside the cavity differ by 2Δ. This can be expressed in the following way:

$$u - \mathrm{i}v = \frac{\overline{f'_s}}{|f'_s|}(|U - \mathrm{i}V|^2 - 2\Delta)^{1/2}.$$

Expressing w'_z in terms of this formula and (3.1), multiplying by f'_s, and substituting $z = f(s)$, we arrive at the expression

$$w'_s = \mathrm{i}\omega\overline{f(s)}f'_s + (|W'_s|^2 - 2\Delta|f'_s|^2)^{1/2}. \tag{3.2}$$

This relation is valid on the dividing streamline between points A and B, or, equivalently, on the corresponding segment $[A_s, B_s]$ of the real axis in the s-plane. However, it cannot be used outside this segment because the right-hand side is not an analytic function of s while the left-hand side is an analytic function (because $w(z)$ and $z = f(s)$ are analytic functions).

The main idea of our approach is to replace the right-hand side of (3.2) with an analytic function taking the same values on $[A_s, B_s]$ as the right-hand side of (3.2). On this segment $s = \bar{s}$, and hence $\overline{f(s)} = \overline{f(\bar{s})}$. From the well-known property of analytic functions $\overline{f(\bar{s})}$ is an analytic function of s. Similarly, we may replace $|f'_s(s)|^2$ with $f'_s(s)\overline{f'_s(\bar{s})}$. Notice now that $W'_s(s)$ is a complex-conjugated velocity of the flow that can be obtained by mapping conformally the flow in the z-plane onto the s-plane. The segment $[A_s, B_s]$ of the real axis is a streamline of this flow, and for this reason W'_s takes real values on it, and, therefore, $|W'_s(s)|^2 = W'^2_z(s) = W'_s(s)\overline{W'_s(\bar{s})} = \overline{W'_s(\bar{s})}^2$. The expression for the vortex flow velocity, which we are deriving, will be used inside the cavity, whereas the potential W is determined outside, that is, on the other side of the real axis. Hence the last of the equivalent representations for $|W'_s(s)|^2$ is the most appropriate. As a result, we obtain

$$w'_s = \mathrm{i}\omega\overline{f(\bar{s})}f'_s(s) + \left(\overline{W'_s(\bar{s})}^2 - 2\Delta f'_s(s)\overline{f'_s(\bar{s})}\right)^{1/2}. \tag{3.3}$$

Finally, multiplying this expression by s'_z and substituting into (3.1) gives the following expression for the complex-conjugated velocity of the vortex flow inside the cavity:

$$u - \mathrm{i}v = \mathrm{i}\omega\left(\overline{f(\bar{s})} - \bar{z}\right) + s'_z(z)\left(\overline{W'_s(\bar{s})}^2 - 2\Delta f'_s(s)\overline{f'_s(\bar{s})}\right)^{1/2}. \tag{3.4}$$

Here, $s = s(z)$ is a function inverse to $f(s)$. Now the streamlines inside the cavity and, hence, the cavity shape can be found by integrating (3.4) numerically.

The uniqueness of the solution obtained does not follow directly from (3.4), but its proof is now so simple that it is not reproduced here.

Returning to the formulation and considering the result, it can be seen in what sense exactly the problem is ill-posed. From (3.4) it follows that the solution varies only slightly if the variation of $W(s)$ is small for all s such that $f(s)$ lies inside the cavity. This will be the case if, for example, the body shape is changed slightly somewhere outside the portion between points A and B. We may say that the problem turns out to be well-posed with respect to variations of the external flow.

However, the formulation of the problem involves $f(s)$ only on $[A_s, B_s]$ whereas (3.4) involves $f(s)$ outside this segment also. As is well known, quite different analytic functions may take very similar values on a segment. For example, on a segment a function can be approximated with any degree of accuracy both with polynomials and with truncated Fourier series but outside the segment these approximations will differ from one another and from the approximated function. Therefore, a small variation in the shape of the dividing streamline may result in a considerable change of the cavity shape. This is why the requirement of prescribing $f(s)$ explicitly and, hence, with absolute accuracy was imposed. It is now clear that this constraint is not mandatory. The function $z = f(s)$ may be given approximately, for example as a two-dimensional array, but in that case it should be given not only on the segment $[A_s, B_s]$ of the real axis but also in the region of the s-plane containing all values of s needed for calculating the flow in the cavity.

Note that if $s(z)$ is considered as a potential of an auxiliary flow in the z-plane then the dividing streamline will be a portion of a streamline of that auxiliary flow. This interpretation simplifies the selection of a suitable $f(s)$. It also demonstrates that prescribing $W(z)$, and, hence, the dividing streamline shape does not determine the cavity shape uniquely. Indeed, there are many different flows with portions of streamlines that coincide. Therefore, even for the shape of the dividing streamline given, $f(s)$ can be varied to some extent.

It is crucially important that an explicit expression for the potential W is not needed for using our result. The reason is that in deriving (3.3) it was possible to use the reflection principle, substituting $\overline{W'(\bar{s})}$ for $W'(s)$, for analytic continuation of W through the cavity boundary. Hence, it is sufficient to calculate the potential of the flow outside the cavity numerically. This is an important advantage over the results of Abrashkin & Yakubovich (1988), who, assuming that $W(z)$ is given explicitly, used the W-plane instead of the auxiliary s-plane introduced in this paper.

Let us consider (3.4) in more detail. Note first that the coordinates of points A and B do not enter (3.4). This was to be expected on the basis of the following well-known property of analytic functions: an analytic function is uniquely determined by its behaviour in an arbitrary small vicinity of a single point. Therefore, if the flow described by (3.4) has a closed streamline, a portion of which coincides with the dividing streamline (hence, this closed streamline is the cavity boundary), then the location of the cavity edges is uniquely determined by $W(z)$ and $f(s)$. It is easily seen that the first term on the right-hand side of (3.4) describes a vortex flow with vorticity equal to ω, and velocity equal to zero on the support curve. The second term on the right-hand side of (3.4) describes the potential flow with one of the streamlines coinciding with the portion of the body surface between A and B and with the velocity squared which is 2Δ smaller on that portion than the square of the velocity of the external potential flow. In particular, for $\Delta = 0$ this flow is an analytic continuation of the external solution across the curve AB. From (3.4) it follows in this case that in the immediate vicinity of the cavity edges the contribution from the vortex component is small. For this reason near these points the cavity wall is a reflection of the body surface from the support curve. For $\Delta > 0$, in the case when the velocity of the internal flow does not equal zero at the cavity edges a similar conclusion can easily be proved by considering the asymptotics of (3.4) near these points. Therefore, the cavity wall branches from the support curve at a point where the body surface branches from this curve. From this result it also follows that the solution is univalent only if the support curve continues inside the body, as shown with a broken curve in figure 1.

The cavity wall can also branch from the support curve at a point where the square of the velocity of the external flow equals 2Δ, and, correspondingly, the velocity of the internal flow equals zero. If the external flow velocity is not singular at that point then the asymptotics of the internal flow near that point follows directly from (3.4). It turns out that in this case the angle between the dividing streamline and the cavity wall equals $2\pi/3$.

Lastly, the cavity wall can branch from the support curve at a point where the vortex flow represented by the first term on the right-hand side of (3.4) has a branch point. This rare case is not considered here.

Therefore, if the given points A and B do not satisfy at least one of these restrictions then the cavity edges will not coincide with them, and the problem will have no solution. However, there can also be no solution for other reasons.

First, it is possible that the cavity wall goes to another plane and crosses the body surface, in other words, the solution may be multi-valued. As already pointed out this is always the case if the support curve continues beyond the body at the cavity edge. However, it must not be ruled out that the calculated cavity wall may cross the body surface far from points A and B.

Second, the calculated cavity may be not closed. The simplest example is the case $f(s) = s$, $W' = \text{const.}$, that is a uniform external flow above a straight surface, a portion of which may be considered as the dividing streamline. The flow in the cavity is then a simple shear flow independent of the longitudinal coordinate.

Third, the flow may have singularities inside the cavity. There are two sources of singularities. In the s-plane the singularities of the potential W are reflected from the real axis giving singularities inside the cavity. For example, if the dividing streamline is a circular arc in the z-plane then at the centre of the circle a dipole is to be expected, which is the reflection of infinity in the external flow. Alternatively, the singularities of the conformal mapping of s onto z also produce singularities of the velocity. To be more specific, as the analysis of (3.4) shows, a singularity may (and probably will) appear at the point $z_0 = f(s_0)$, if at this point s is singular as a function of z or if $f(s)$ is singular at $s = \overline{s_0}$. The dividing streamline can be approximated with very different functions $f(s)$. This freedom may be used for eliminating singularities of the velocity inside the cavity. In figure 1 the support curve is a parabola, and its focus, which is marked with a star, is a singular point of the velocity field (3.4). Interestingly, there is a certain distortion of the cavity wall near this point. Note that if the singularity inside the cavity is a branch point then the solution is multi-valued.

Two simple hints may be given concerning the dependence of the cavity shape on the parameters ω and Δ. Let us suppose that the velocity of the external potential flow has a maximum U_{max} at some internal point on the dividing streamline. Let 2Δ be only slightly less than U_{max}^2. Then at a small distance in each direction from the maximum there are two points on the dividing streamline at which the square of the velocity of the external flow equals 2Δ. Accordingly, the velocity of the internal flow equals zero at these points. When the cavity wall branches from the support curve at stagnation points of the internal flow the cavity edges coincide with the points at which the absolute values of the external flow velocity are equal. A decrease in Δ results in an increase in the cavity length, because these points move apart.

If the vorticity ω tends to infinity then far from points A and B the main term of the velocity is $u_\tau = -\omega n + h/2$, where u_τ is the velocity component tangential to the dividing streamline, n is the coordinate along the normal to the dividing streamline, directed from the cavity, and h is the cavity depth. Since the velocity on the cavity boundary is of order one then $h \sim 1/\omega$ for $\omega \to \infty$.

Expression (3.4) gives an exact solution in the case when the dividing streamline is given explicitly. In practice the body and, hence, the dividing streamline would often be given by a set of points. Through a finite number of fixed points an infinite number of explicit analytic curves (that is, curves like a support curve) can be drawn. (Note also, that an exact body shape may be non-analytic or it may be a combination of several portions of different analytic curves. Accordingly, outside the dividing streamline the support curve need not coincide with the body contour.) As the problem is ill-posed, quite different cavities may result in almost identical shapes of the dividing streamline. For this reason the exact shape of the cavity is not defined uniquely or even approximately in the case when the dividing streamline is given by a set of points. Hence, the difference between the cavity shape obtained (for example, in a numerical calculation) and an exact solution cannot be an accuracy measure. Therefore, the only reasonable measure of the accuracy of the result is the difference between the dividing streamline obtained and the points given.

Naturally, depending on the accuracy required, the support curve may be drawn not through the given points on the dividing streamline but sufficiently close to them as is in fact the case in the following section.

4. Numerical calculation

To check the method and for illustration purposes, and also for future applications, a program was written which calculates the flow past an airfoil with a cavity. The airfoil shape is prescribed in the following way. First, several points on the airfoil must be given, numbered clockwise: $z_0, z_1, \ldots, z_{n-1}, z_n$ in such a way that $z_0 = z_n$ is the sharp edge. Then the airfoil shape is assumed to be described by a cubic spline. More specifically, each airfoil segment between points z_{l-1}, z_l is described in the parametric form and

$$z = S_l(p) = m_{l-1}(1-p)^3 + m_l p^3 + (z_{l-1} - m_{l-1})(1-p) + (z_l - m_l)p,$$
$$0 \leqslant p \leqslant 1, \quad l = 1, \ldots, n.$$

Here, the coefficients satisfy the system of equations

$$\begin{aligned} m_0 &= -m_n, \\ m_{l-1} + 4m_l + m_{l+1} &= z_{l-1} - 2z_l + z_{l+1}, \quad l = 1, \ldots, n-1, \\ 2m_n - 2m_0 + m_{n-1} - m_1 &= z_{n-1} - z_1. \end{aligned}$$

This ensures that the airfoil surface is smooth and that its curvature is continuous. The trailing edge of the airfoil forms a cusp so that at the edge the velocity is not zero, and the curvatures of the upper and lower airfoil surfaces have equal values and different signs. The condition at the trailing edge can be easily modified, and some calculations, not presented here, were performed for airfoils with a finite angle at the edge. This method of defining the airfoil shape is quite flexible and makes it possible to describe easily airfoils of various shapes.

Then one of the segments of this airfoil is replaced with an analytic support curve $z = f(s)$. The specific form of this function is determined in a subroutine and can be easily varied. In particular, flows in figures 1–3 were calculated for

$$z = f(s) = 2(z_k - 2a + z_{k+1})s^2 + (-3z_k + 4a - z_{k+1})s + z_k, \tag{4.1}$$

where

$$a = S_k(0.5 + \mathrm{i}\delta). \tag{4.2}$$

Here, k is the number of the substituted segment and δ is a complex parameter. This expression describes a parabola passing through the segment ends and the point $z = a$. In some other calculations the support curve was a hyperbola or a curve described by an equation of the form $z = C_1 \log(s - \delta) + C_2$.

Each segment, including the dividing streamline, was then represented by a number of vortex panels for the calculation of the external potential flow. This flow was calculated using a version of the boundary-element method described by Kuethe &

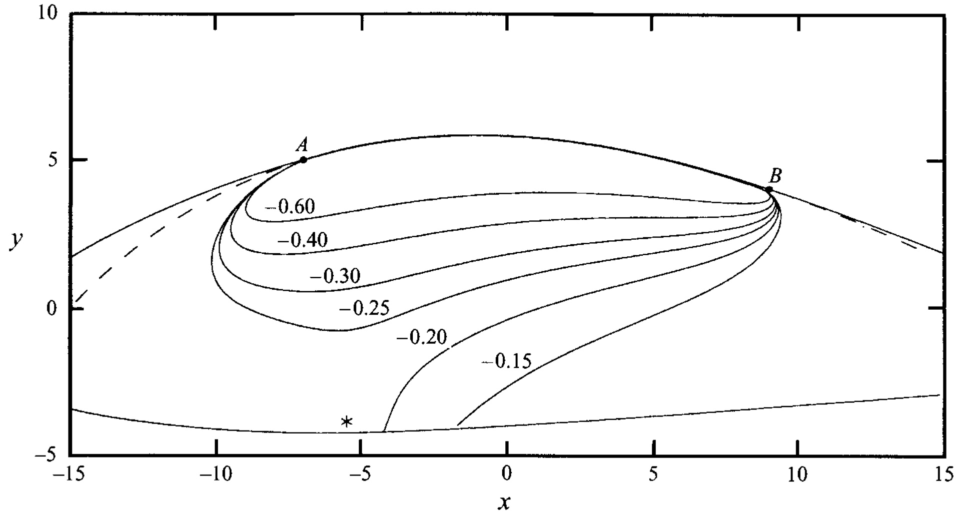

FIGURE 2. Cavity shapes for different ω, $\Delta = 1.1$. The values of ω are given near the curves. For $\omega = -0.15$ and -0.20 the cavity is not closed.

Chow (1986), except that for solving the corresponding system of algebraic equations a more efficient Gauss method was used instead of the Cramer rule. The resulting solution satisfies the Kutta–Joukowski condition. The program described by Kuethe & Chow (1986) calculates the velocity distribution on the profile but modifying it for calculating the velocity in the entire flow field is so easy that it is not described here.

The velocity inside the cavity can now be calculated from (3.4). The cavity shape was calculated approximately as a streamline passing very close to the cavity boundary. The calculations were performed in the s-plane using the second-order predictor–corrector method, starting from the point $s = s_0 - i\epsilon$ with a very small ϵ. We cannot put $\epsilon = 0$ because the cavity edges are singularities of the velocity field and the streamlines branch there. In most of the calculations the values $s_0 = 0.5$ and $\epsilon = 0.001$ were used.

The accuracy of the calculations depends on the number of panels used in the boundary-element method, the integration step in the calculation of the cavity shape, and ϵ. The program for calculating the potential flow was checked by comparison with the results of calculating flows past airfoils by other methods. The test calculations of the cavity wall shape were carried out with different integration step sizes. For sufficiently small steps the dependence of the results on the square of the integration step was found to be linear. This agrees with the second order of accuracy of the method. Calculations were performed for different ϵ, and it was shown that for small ϵ the results depend on ϵ linearly. The information obtained in the test calculations was used to ensure that the error in figures 1–3 is within the graphical accuracy in the sense that further increase in the number of panels, decrease in the integration step, and decrease in ϵ do not cause changes in the figures that could be seen with the naked eye. More detailed description of the test calculations is not necessary because the methods of our numerical calculations were, in fact, quite traditional.

The advantage of our method is that varying Δ and ω does not require additional computations of the external potential flow. Moreover, neither the potential flow computation nor integration along a streamline inside the cavity need large computer resources.

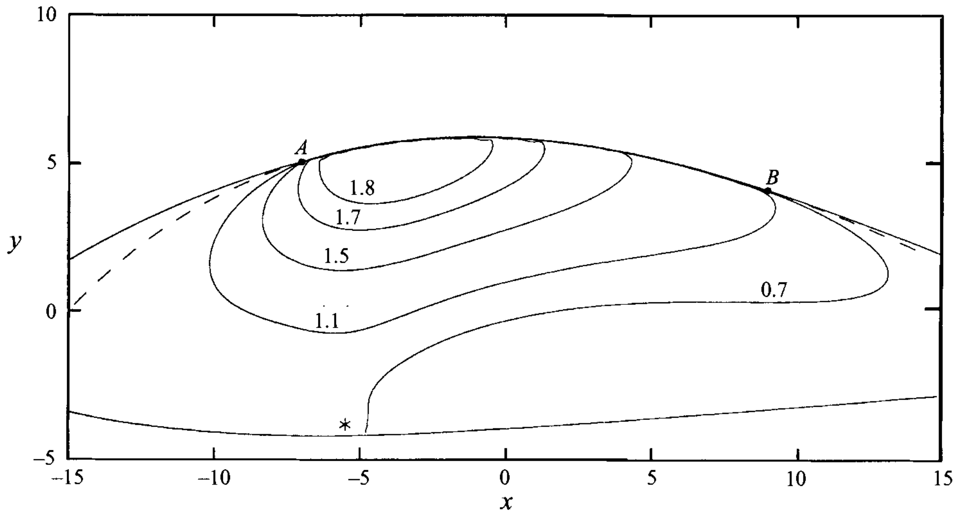

FIGURE 3. Cavity shapes for different Δ, $\omega = -0.25$. The values of Δ are given near the curves. For Δ = 0.7 the cavity is not closed.

Calculations were carried out for a fixed airfoil and dividing streamline shape but for various Δ and ω in an attempt to study the influence of these parameters on the cavity shape. The longitudinal size of the closed streamlines region depends on the vorticity only weakly. This is also true in the case when the position of the cavity edges is fixed by singularities as was discussed in the previous section. An increase in the absolute value of vorticity decreases the cavity depth. An increase in Δ reduces the size of the cavity while its length-to-depth ratio remains approximately constant. These results are illustrated in figures 2 and 3 where the cavity shapes for various Δ and ω are given. Such results may depend on the airfoil and dividing streamline shape so that they must be used with caution.

The integration along a streamline was always started between points A and B and continued until the streamline was closed, but if the cavity wall crossed the airfoil surface the integration had to be interrupted.

The airfoil in figures 1–3 is determined by the following points z_i : (31, −4), (14, −3), (0, −4), (−11, −4), (−18, −1), (−7, 5), (9, 4). The support curve is determined by (4.1) and (4.2) with $k = 6$ and $\delta = 0.01$.

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